**Enrollment No: _____ Exam Seat No:_____**

# C.U.SHAH UNIVERSITY
## Summer Examination-2019

**Subject Name: Discrete Mathematics**

**Subject Code: 4TE04DSM1** **Branch: B.Tech (CE)**

**Semester: 4** **Date: 15/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

**Instructions:**

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

---

**Q-1 Attempt the following questions:** **(14)**

**a)** Find the least and greatest element in the poset $\langle N, D \rangle$, if they exist. **(02)**

**b)** Define: Poset, Pseudo Graph **(02)**

**c)** State Pigeonhole principle. **(02)**

**d)** Find the atom and anti-atom of $\langle S_{60}, D \rangle$. **(02)**

**e)** Prove that $(ab + ab')a'b' = 0$. **(02)**

**f)** $(Z_{11}, +_{11})$ is cyclic group.- True or False? **(01)**

**g)** How many edges are there in a graph with 7 vertices each of degree 4? **(01)**

**h)** Define: Difference of Fuzzy set **(02)**

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions.** **(14)**

**a)** State and prove Stone's representation theorem. **(10)**

**b)** State Distributive law for fuzzy subsets and prove any one. **(04)**

**Q-3 Attempt all questions** **(14)**

**a)** Show that $\{0, 2, 4, 6\}$ is a subgroup of $(Z_8, +_8)$, where $+_8$ is addtion modulo 8. **(05)**

**b)** Prove that $\langle S_{42}, D \rangle$ is a complemented lattice and also draw the Hasse diagram of it. **(05)**

**c)** Prove that $\langle S_6, D \rangle$ is a sub lattice of $\langle S_{30}, D \rangle$ . **(04)**

**Q-4 Attempt all questions** **(14)**

**a)** Show that the set Q \ {-1} is an abelian group with respect to the binary operation $a * b = a + b + ab$, for all a, b $\in$ G. **(07)**

**b)** Let $E=\{a,b,c\}$, $\underset{\sim}{A}=\{(a,0.4),(b,0.7),(c,0.6)\}$, $\underset{\sim}{B}=\{(a,0.8),(b,0.2),(c,0.5)\}$ then find the following: **(07)**

1) $\underset{\sim}{A}\cup\underset{\sim}{B}$ 2) $\underset{\sim}{A}\cdot\underset{\sim}{B}$ 3) $\underset{\sim}{A}\hat{+}\underset{\sim}{B}$ 4) $\underset{\sim}{A}-\underset{\sim}{B}$ 5) $\underset{\sim}{A}\cap\underset{\sim}{B}$ 6) $\underset{\sim}{A}'$ 7) $\underset{\sim}{B}'$

**Q-5 Attempt all questions** **(14)**

**a)** For a lattice $\langle P(\{a,b,c\}),\subseteq\rangle$, answer the following questions: **(07)**

i) Find cover of each element and draw the Hasse diagram.

ii) Find lower bound, upper bound, greatest lower bound, least upper bound of $A=\{\{a,b\}\}$.

iii) Find the least and greatest element of it.

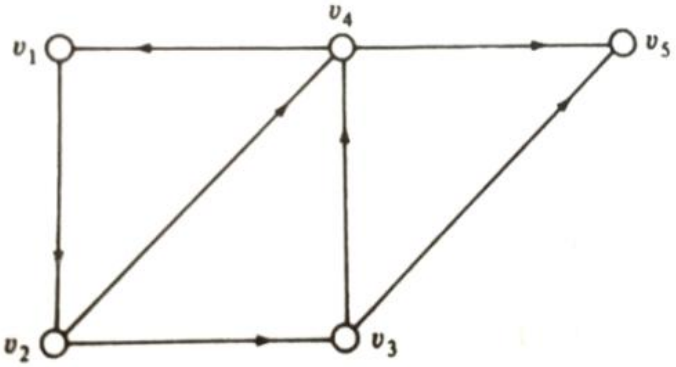
**b)** Let $\langle L,\leq\rangle$ be a lattice $a,b\in L$ then prove that **(07)**

*i)* $a\leq b\Leftrightarrow a*b=a\Leftrightarrow a\oplus b=b$ *ii)* $a\leq c\Leftrightarrow a\oplus(b*c)\leq(a\oplus b)*c$

**Q-6 Attempt all questions** **(14)**

**a)** i) Draw the graph represented by given adjacency matrix $\begin{bmatrix}1&2&2&1\\1&0&1&2\\2&1&1&0\\0&2&1&0\end{bmatrix}$. **(05)**

ii) Write the adjacency matrix from the given digraph.




**b)** State and prove Lagrange's theorem. **(05)**

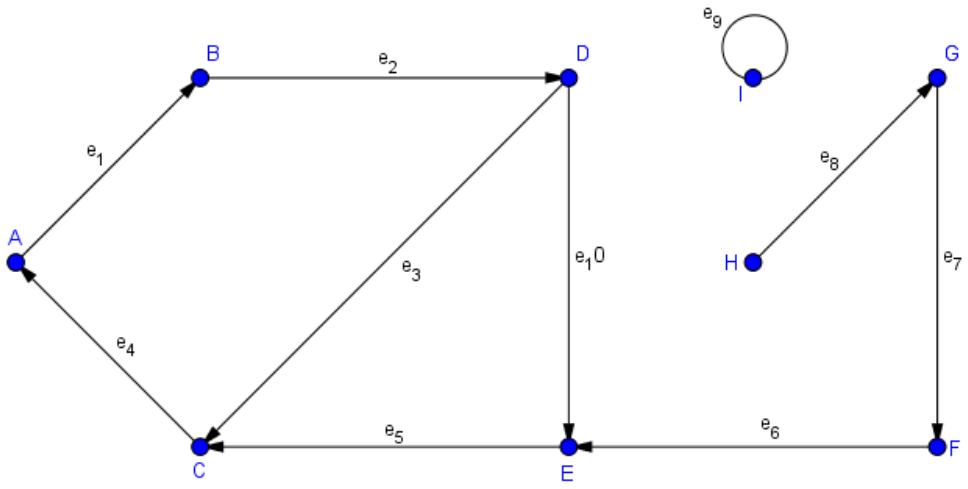
**c)** By using mathematical induction prove that $1+2+3+...+n=\dfrac{n(n+1)}{2}$. **(04)**

**Q-7 Attempt all questions.** **(14)**

**a)** Obtain the sum of product canonical form of the Boolean expression in three variables $\alpha(x,y,z)=(x\oplus y)'\oplus z$. **(05)**

**b)** Prove that $(Z_6^*,\times_6)$ is a group. Is it commutative? **(05)**

**c)** Find all node base of the following diagraph shown in the figure. **(04)**




**Q-8 Attempt all questions.** **(14)**

**a)** Define tree and draw a directed tree from following and also find the representation of binary tree. $\left(v_0\left(v_1\left(v_2\right)\left(v_3\left(v_4\right)\left(v_5\right)\right)\right)\left(v_6\left(v_7\left(v_8\right)\right)\left(v_9\right)\left(v_{10}\right)\right)\right)$ **(07)**

**b)** Do as directed: **(07)**

1) Translate the following in your own words.

$A(x)$: $x$ is a whale, $B(x)$: $x$ is a fish, $C(x)$: $x$ lives in water.

i) $(\exists x)\left(B(x)\wedge \sim A(x)\right)$ ii) $(\forall x)\left(A(x)\vee C(x)\right)\Rightarrow B(x)$

2) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, $n \geq 2$; $a_0 = 1$, $a_1 = 2$.